

Lightest Higgs Boson Mass in Split Supersymmetry with See-saw Mechanism

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In the minimal supersymmetric standard model extended by including right-handed neutrinos with see-saw mechanism, the neutrino Yukawa couplings can be as large as the top-quark Yukawa couplings and thus the neutrino/sneutrino may cause sizable effects in Higgs boson self-energy loops. Our explicit one-loop calculations show that the neutrino/sneutrino effects may have an opposite sign to top/stop effects and thus lighten the lightest Higgs boson. If the soft-breaking mass of the right-handed neutrino is very large (at the order of Majorana mass scale), such as in the split-SUSY scenario, the effects can lower the lightest Higgs boson mass by a few tens of GeV. So the Higgs mass bound of about 150 GeV in split-SUSY may be lowered significantly if right-handed neutrinos come into play with see-saw mechanism.

14.80.Cp, 14.80.Ly, 12.60.Jv

Searching for the Higgs boson is a primary goal of ongoing and forthcoming high energy colliders. The verification or disproof of the existence of a light Higgs boson will serve as an acid test for both the Standard Model (SM) and the supersymmetry (SUSY) extensions. While for the SM a light Higgs boson is favored indirectly by the precision electroweak data, the Minimal Supersymmetric Standard Model (MSSM) predicts a Higgs boson (h) lighter than about 130 GeV [1-3]. In the recently proposed split-SUSY scenario [4], where the fine-tuning problem is argued not to be solved by SUSY and thus the soft-breaking masses of sfermions can be very large, such a bound is relaxed to about 150 GeV. It is well known that these bounds, much higher than the tree-level prediction $m_h < m_Z$, are obtained by considering loop effects, mainly the top/stop loop effects. The underlying reason for the sizable top/stop loop effects is the largeness of the top-quark Yukawa couplings. So, if any other particles have large Yukawa couplings, their loop effects in the Higgs boson mass should also be taken into account.

The recent compelling evidence of neutrino oscillation indicates massive neutrinos, which necessitates the introduction of right-handed neutrino superfields [5] in the MSSM. If the Majorana masses of the right-handed neutrinos are large enough, the tiny masses of light neu-

trinos can be naturally obtained, which is the so-called see-saw mechanism [6]. In such a supersymmetric see-saw model the neutrino Yukawa couplings can be as large as the top-quark Yukawa couplings [7]. Thus, the neutrino/sneutrino may cause sizable loop effects in Higgs boson masses. In this note we focus on such effects and perform an explicit one-loop calculations¹.

We start our calculation from the superpotential. Compared with the MSSM, the supersymmetric see-saw model contains the additional right-handed neutrino superfield in the superpotential

$$-\frac{1}{2}\nu_R^c M_R \nu_R^c + \nu_R^c \mathbf{Y}_\nu L \cdot H_2, \quad (1)$$

where Y_ν is the Yukawa couplings of neutrinos and M_R is the Majorana mass, both of which are generally 3×3 matrices in the generation space. For simplicity and illustration, we in our analysis ignore the flavor structure and consider only one generation of neutrinos². Such an addition of right-handed neutrino superfields will enrich the phenomenology of both neutrino sector and sneutrino sector.

First, we examine the neutrino sector. The neutrino mass term is given by

$$(\nu_L, \nu_R^c) \begin{pmatrix} 0 & Y_\nu v_2 \\ Y_\nu v_2 & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + h.c.$$

¹Note that there are two approaches for calculating such leading loop effects. One is the explicit loop calculations used in this work. The other is the renormalization group technique. We note that in the approach of explicit loop calculations, one-loop calculations may be not accurate enough and thus higher order loop effects may need to be considered. But in this letter we only perform one-loop calculations in order to illustrate the possible size of the effects. In our future work we will provide more complete calculations, either by considering higher order loops or by the renormalization group to resum the leading logs.

²The general result with the three families of neutrinos is quite complicated and will be presented elsewhere [8].

$$= -(\nu_1, \nu_2) \begin{pmatrix} \frac{Y_\nu^2 v_2^2}{M_R} & 0 \\ 0 & M_R(1 + \frac{Y_\nu^2 v_2^2}{M_R^2}) \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} + h.c. \quad (2)$$

Here v_2 is the vacuum expectation value of the Higgs doublet H_2 and the mass matrix is diagonalized by an unitary rotation, which rotates ν_L and ν_R^c into two mass eigenstates $\nu_{1,2}$, i.e.,

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_\nu & \sin \theta_\nu \\ -i \sin \theta_\nu & i \cos \theta_\nu \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \quad (3)$$

with the mixing angle θ_ν determined by

$$\tan(2\theta_\nu) = -\frac{2Y_\nu v_2}{M_R}. \quad (4)$$

Note that all these states are the two-component spinors. We can define the four-component Majorana spinors as $\nu_1 \equiv (\nu_1, \bar{\nu}_1)^T$ and $\nu_2 \equiv (\nu_2, \bar{\nu}_2)^T$, which will be used in our following calculations.

Then the neutrino Yukawa couplings with neutral Higgs bosons are given by

$$\frac{1}{2\sqrt{2}} \left[(H \sin \alpha + h \cos \alpha) \bar{\nu}_i (\xi_{ij} P_L + \xi_{ij}^\dagger P_R) \nu_j + i \cos \beta A \bar{\nu}_i (\xi_{ij} P_L - \xi_{ij}^\dagger P_R) \nu_j \right], \quad (5)$$

where α is the mixing angle between the two neutral CP-even Higgs bosons, $\tan \beta = v_2/v_1$ is the ratio of the vacuum expectation values of the two Higgs doublets, $P_{L,R} \equiv (1 \mp \gamma_5)/2$, and ξ_{ij} are given by

$$\xi = \begin{pmatrix} \sin 2\theta_\nu Y_\nu & -i \cos 2\theta_\nu Y_\nu \\ -i \cos 2\theta_\nu Y_\nu & \sin 2\theta_\nu Y_\nu \end{pmatrix}. \quad (6)$$

Now we turn to the sneutrino sector. The sneutrino mass terms arise from F-terms, D-terms as well as the soft-breaking terms given by

$$V_{soft} = \tilde{L}^* m_L^2 \tilde{L} + \tilde{\nu}_R^{c*} m_{\tilde{\nu}_R}^2 \tilde{\nu}_R^c + B^* \tilde{\nu}_R^{c*} M_R^* \tilde{\nu}_R^c + B \tilde{\nu}_R^c M_R \tilde{\nu}_R^c - A_\nu \epsilon_{\alpha\beta} H_2^\alpha \tilde{\nu}_R^c Y_\nu \tilde{L}^\beta - A_\nu^* \epsilon_{\alpha\beta} H_2^* \tilde{\nu}_R^{c*} Y_\nu^* \tilde{L}^{*\beta}. \quad (7)$$

Throughout this paper, we assume $M_R > B, A_\nu$ and $M_R \gg v_2, m_Z$, and treat the soft terms proportional to B and A_ν as interactions. Then the sneutrino mass terms are given by [9]

$$\begin{pmatrix} \tilde{\nu}_L^*, \tilde{\nu}_R^{c*} \end{pmatrix} \begin{pmatrix} \tilde{M}_{LL}^2 & M_R Y_\nu v_2 \\ M_R Y_\nu v_2 & \tilde{M}_{RR}^2 \end{pmatrix} \begin{pmatrix} \tilde{\nu}_L \\ \tilde{\nu}_R^c \end{pmatrix} \approx \begin{pmatrix} \tilde{\nu}_1^*, \tilde{\nu}_2^* \end{pmatrix} \begin{pmatrix} m_L^2 & 0 \\ 0 & M_R^2 + m_{\tilde{\nu}_R}^2 \end{pmatrix} \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \end{pmatrix}, \quad (8)$$

where

$$\tilde{M}_{LL}^2 = m_L^2 + Y_\nu^2 v_2^2 + \frac{1}{2} m_Z^2 \cos 2\beta, \quad (9)$$

$$\tilde{M}_{RR}^2 = M_R^2 + Y_\nu^2 v_2^2 + m_{\tilde{\nu}_R}^2. \quad (10)$$

The sneutrino mass matrix is diagonalized by an unitary rotation in Eq.(8) and the two mass eigenstates $\tilde{\nu}_{1,2}$ are given by

$$\begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{\tilde{\nu}} & \sin \theta_{\tilde{\nu}} \\ -\sin \theta_{\tilde{\nu}} & \cos \theta_{\tilde{\nu}} \end{pmatrix} \begin{pmatrix} \tilde{\nu}_L \\ \tilde{\nu}_R^c \end{pmatrix} \quad (11)$$

with the mixing angle $\theta_{\tilde{\nu}}$ determined by

$$\sin(2\theta_{\tilde{\nu}}) = \frac{2Y_\nu M_R v_2}{m_{\tilde{\nu}_1}^2 - m_{\tilde{\nu}_2}^2}. \quad (12)$$

Then the sneutrino Yukawa couplings are given by

$$\begin{aligned} & -\frac{Y_\nu}{\sqrt{2}} \{ (2Y_\nu v_2 + \sin 2\theta_{\tilde{\nu}} M_R) (H \sin \alpha + h \cos \alpha) \tilde{\nu}_1^* \tilde{\nu}_1 \\ & + (2Y_\nu v_2 - \sin 2\theta_{\tilde{\nu}} M_R) (H \sin \alpha + h \cos \alpha) \tilde{\nu}_2^* \tilde{\nu}_2 \\ & + \cos 2\theta_{\tilde{\nu}} M_R (H \sin \alpha + h \cos \alpha) (\tilde{\nu}_1^* \tilde{\nu}_2 + \tilde{\nu}_2^* \tilde{\nu}_1) \\ & - i M_R \cos \beta A (\tilde{\nu}_1^* \tilde{\nu}_2 - \tilde{\nu}_2^* \tilde{\nu}_1) \\ & + \frac{Y_\nu}{\sqrt{2}} [(H \sin \alpha + h \cos \alpha)^2 \\ & + A^2 \cos^2 \beta] (\tilde{\nu}_1^* \tilde{\nu}_1 + \tilde{\nu}_2^* \tilde{\nu}_2) \}. \end{aligned} \quad (13)$$

In addition, the bilinear and trilinear interactions from the soft-breaking terms are given by

$$\begin{aligned} & -B(\sin \theta_{\tilde{\nu}} \tilde{\nu}_1 + \cos \theta_{\tilde{\nu}} \tilde{\nu}_2) M_R (\sin \theta_{\tilde{\nu}} \tilde{\nu}_1 + \cos \theta_{\tilde{\nu}} \tilde{\nu}_2) \\ & -A_\nu Y_\nu (\sin \theta_{\tilde{\nu}} \tilde{\nu}_1 + \cos \theta_{\tilde{\nu}} \tilde{\nu}_2) (\cos \theta_{\tilde{\nu}} \tilde{\nu}_1 - \sin \theta_{\tilde{\nu}} \tilde{\nu}_2) \\ & \times \left(v_2 + \frac{1}{\sqrt{2}} (H \sin \alpha + h \cos \alpha + i A \cos \beta) \right) + h.c. \end{aligned} \quad (14)$$

The neutrino/sneutrino contribute to Higgs boson self-energies through the diagrams shown in Fig.1. The renormalized self-energies are given by

$$\hat{\Sigma}_h \approx -\omega_\nu \cos^2 \alpha, \quad (15)$$

$$\hat{\Sigma}_H \approx -\omega_\nu \sin^2 \alpha, \quad (16)$$

$$\hat{\Sigma}_h H \approx -\omega_\nu \sin \alpha \cos \alpha, \quad (17)$$

where ω_ν can be decomposed as

$$\omega_\nu = \omega_\nu^{SUSY} + \omega_\nu^{soft} \quad (18)$$

with ω_ν^{SUSY} and ω_ν^{soft} representing respectively the contribution from supersymmetric part and soft-breaking part given by

$$\begin{aligned} \omega_\nu^{SUSY} &= \frac{Y_\nu^2 v_2^2}{4\pi^2} \left[\frac{1}{2} \ln \frac{m_{\tilde{\nu}_1}^2 m_{\tilde{\nu}_2}^2}{M_R^4} - 1 \right. \\ &+ \frac{M_R^2}{m_{\tilde{\nu}_1}^2 - m_{\tilde{\nu}_2}^2} \ln \frac{m_{\tilde{\nu}_1}^2}{m_{\tilde{\nu}_2}^2} + \frac{M_R^4}{(m_{\tilde{\nu}_1}^2 - m_{\tilde{\nu}_2}^2)^2} \\ &\times \left(1 - \frac{m_{\tilde{\nu}_1}^2 + m_{\tilde{\nu}_2}^2}{2(m_{\tilde{\nu}_1}^2 - m_{\tilde{\nu}_2}^2)} \ln \frac{m_{\tilde{\nu}_1}^2}{m_{\tilde{\nu}_2}^2} \right) \Big], \quad (19) \\ \omega_\nu^{soft} &= -\frac{Y_\nu^2}{16\pi^2} \frac{B^2 M_R^2}{m_{\tilde{\nu}_2}^2} \left[\frac{4}{3m_{\tilde{\nu}_2}^2} (2Y_\nu v_2 - \sin 2\theta_{\tilde{\nu}} M_R)^2 \right. \end{aligned}$$

$$\begin{aligned}
& -2 \sin^2 2\theta_{\tilde{\nu}} \frac{M_R^2}{(m_{\tilde{\nu}_2}^2 - m_{\tilde{\nu}_1}^2)^3} \\
& \times \left(m_{\tilde{\nu}_2}^4 - m_{\tilde{\nu}_1}^4 - 2m_{\tilde{\nu}_2}^2 m_{\tilde{\nu}_1}^2 \ln \frac{m_{\tilde{\nu}_1}^2}{m_{\tilde{\nu}_2}^2} \right) \\
& - \frac{Y_\nu^2}{16\pi^2} \frac{M_R^2}{(m_{\tilde{\nu}_2}^2 - m_{\tilde{\nu}_1}^2)^3} (2B^2 M_R^2 \sin^2 2\theta_{\tilde{\nu}} \\
& + 4BM_R A_\nu v_2 \sin 2\theta_{\tilde{\nu}} + A_\nu^2 v_2^2) \\
& \times \left[2(m_{\tilde{\nu}_1}^2 - m_{\tilde{\nu}_2}^2) - (m_{\tilde{\nu}_1}^2 + m_{\tilde{\nu}_2}^2) \ln \frac{m_{\tilde{\nu}_1}^2}{m_{\tilde{\nu}_2}^2} \right] \quad (20)
\end{aligned}$$

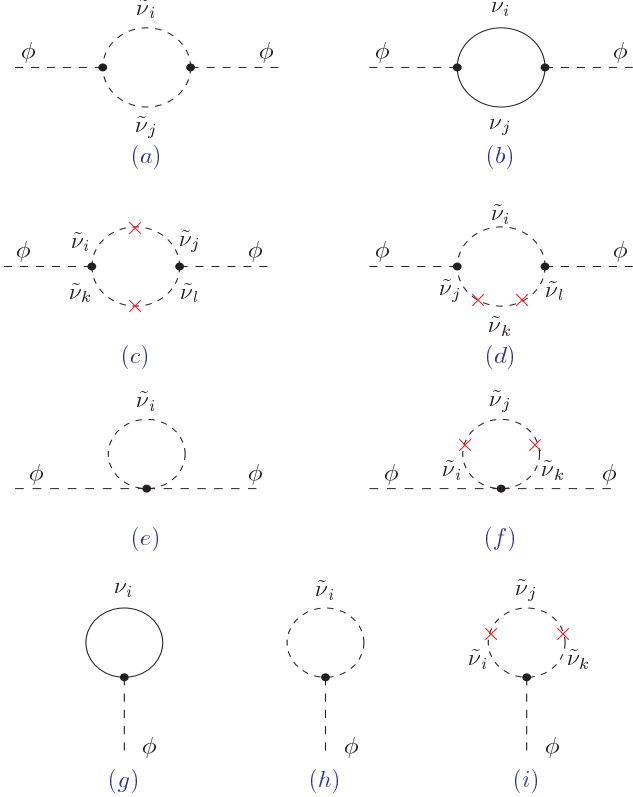


FIG. 1. Feynman diagrams of neutrino/sneutrino effects in Higgs boson self-energies. ϕ denotes a neutral Higgs boson, i.e., h , H or A . The symbol 'x' means a soft-term mass insertion.

In the calculation of ω_ν^{soft} , we dropped the terms suppressed by $v_2^2/m_{\tilde{\nu}_R}^2$. We also checked that the contributions from higher order mass insertions are negligible.

The lightest Higgs boson mass with one-loop neutrino/sneutrino effects is given by [10]

$$\begin{aligned}
m_h^2 = & \frac{M_A^2 + M_Z^2 + \omega_\nu}{2} - \left[\frac{(M_A^2 + M_Z^2)^2 + \omega_\nu^2}{4} \right. \\
& \left. - M_A^2 M_Z^2 \cos^2 2\beta + \frac{\omega_\nu \cos 2\beta}{2} (M_A^2 - M_Z^2) \right]^{1/2} \quad (21)
\end{aligned}$$

From Eq.(20) and Eq.(8), we see that the relevant SUSY parameters are $A_\nu, B, m_{\tilde{L}}, m_{\tilde{\nu}_R}$ and M_R . M_R must be

much larger than m_Z in order to drive the see-saw mechanism, but the scales of the soft-breaking parameters are not clear³. If one insists on naturalness requirement, these soft breaking parameters should not be much larger than the electroweak scale [11]. In this case, since the SUSY breaking in neutrino/sneutrino sector is not significant, the neutrino/sneutrino loop effects on the Higgs boson mass are small, as will be shown in our numerical results. In our numerical calculations we abandon the naturalness, as in the split-SUSY scenario, and thus the soft-breaking masses for sneutrinos can be much larger.

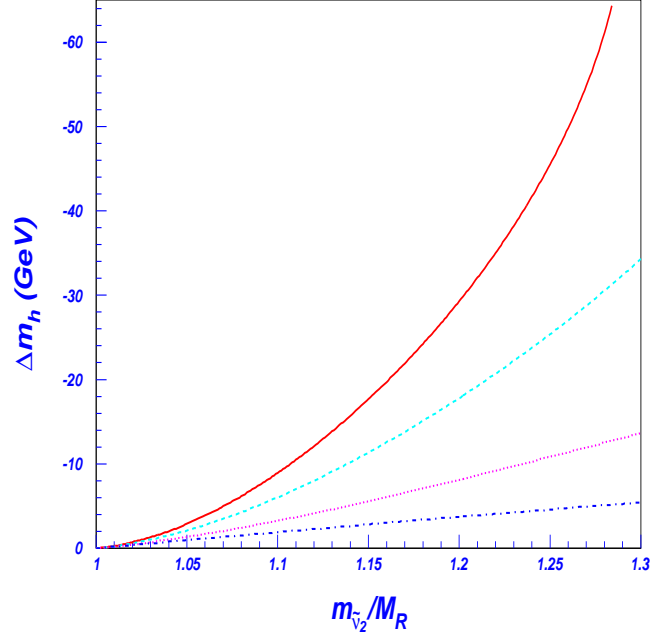


FIG. 2. The neutrino/sneutrino contribution to m_h : the solid, dashed, dotted and dash-dotted curve (from up to down) is for $m_{\tilde{L}} = 10^3, 10^7, 10^{11}, 10^{13}$ GeV, respectively. $M_R = 10^{14}$ GeV and other parameters are fixed in the context.

In Fig. 2 we show the neutrino/sneutrino contribution to m_h , i.e., $\Delta m_h \equiv m_h - m_h^{\text{tree}}$, versus $m_{\tilde{\nu}_2}/M_R$ for $M_R = 10^{14}$ GeV and different values of $m_{\tilde{L}}$. Other parameters are fixed as $m_A = 200$ GeV, $\tan \beta = 30$, $Y_\nu = 1$ and $B = A_\nu = 0$. With above choice of parameters, only ω_ν^{SUSY} contribute to our numerical results.

Since $m_{\tilde{\nu}_2}^2 \approx M_R^2 + m_{\tilde{\nu}_R}^2$ and $m_{\nu_2} \approx M_R$, we see from Fig. 2 that the effects of neutrino/sneutrino are sensitive to the mass splitting of $\tilde{\nu}_2$ (heavy sneutrino) and ν_2 (heavy neutrino). Obviously, such a mass splitting is determined by the right-handed neutrino soft-breaking

³Note that $M_R, m_{\tilde{\nu}_R}$ and B are associated with the $SU(2) \times U(1)$ singlet superfield ν_R and thus may have fundamentally different origin from A_ν and $m_{\tilde{L}}$.

mass $m_{\tilde{\nu}_R}$. So the effects of neutrino/sneutrino depend on the size of the soft masses $m_{\tilde{\nu}_R}$ and $m_{\tilde{L}}$.

Fig. 2 shows that if $m_{\tilde{\nu}_R}$ is as large as M_R , the effects can be quite significant, lowering m_h by a few tens of GeV. If $m_{\tilde{\nu}_R}$ is far smaller than M_R , e.g., of the order of the electroweak scale, then the effects are negligibly small. This can be understood from $h\tilde{\nu}_1\tilde{\nu}_1^*$ interactions shown in Eq.(13): for $m_{\tilde{\nu}_R} \ll M_R$ we have $m_{\tilde{\nu}_2} \simeq M_R$ and then the coupling tends to be zero. As a result, the first term in ω_ν^{SUSY} is cancelled by other terms. When $m_{\tilde{\nu}_R}$ becomes large, such cancellation is alleviated. Another way to understand the above results is that for soft-breaking parameters of electroweak scale, the SUSY-breaking in neutrino/sneutrino sector is not significant and thus the contributions from neutrino loops tend to be cancelled out by those from sneutrino loops.

In case of large SUSY breaking in neutrino/sneutrino sector, the neutrino/sneutrino loop effects are proportional to $\log(m_{\tilde{\nu}_1}/m_{\tilde{\nu}_2}) \approx \log(m_{\tilde{L}}/\sqrt{M_R^2 + m_{\tilde{\nu}_R}^2})$. A smaller $m_{\tilde{L}}$ leads to a larger splitting between $m_{\tilde{\nu}_1}$ and $m_{\tilde{\nu}_2}$ and thus gives a larger contribution.

We found that the results are not sensitive to $\tan\beta$ and m_A . Note that we fixed $B = A_\nu = 0$ and thus ω_ν^{soft} vanished in Fig. 2. If we choose a non-zero B or A_ν , we find that the contribution of ω_ν^{soft} to m_h is also negative, whose magnitude depends on the size of B and A_ν . For $B \ll M_R$ and $A_\nu \ll M_R$, the contribution of ω_ν^{soft} is negligible. But if any one, B or A_ν , is as large as M_R , ω_ν^{soft} can lower m_h by a few tens of GeV, regardless of the magnitude of $m_{\tilde{\nu}_R}$.

Since the effects depend sensitively on the right-handed neutrino soft-breaking masses, it is important to know how large they possibly are. Unfortunately, their origin is not clear and thus their values are theoretically arbitrary so far. In the split-SUSY scenario [4], the soft-breaking masses of sfermions (and thus $m_{\tilde{\nu}_R}$) can be very large. Therefore, the neutrino/sneutrino loop effects on Higgs boson mass may be sizable in split-SUSY if right-handed neutrinos are introduced with see-saw mechanism.

Including such large negative effects, the upper bound on the lightest Higgs boson mass of about 150 GeV in split-SUSY [4] will be lowered significantly. The LEP experiment already set a lower bound of about 90 GeV [12] on the SUSY Higg mass and the Fermilab Tevatron collider will further push up the lower bound in case of unobservation. So the lightest Higgs boson mass will be a crucial test for various SUSY models.

We note that the large Yukawa couplings of neutrinos or large soft-masses of right-handed neutrinos will not cause any problems in the precision electroweak fit of the SM, such as the parameterized variables S , T and U . The reason is that the right-handed neutrinos or sneutrinos have no gauge couplings. The left-handed sneutrinos contribute to S , T and U in a similar way as other sfermions and their effects decouple as they get heavy.

We conclude that due to the unsuppressed neutrino Yukawa couplings in SUSY see-saw model, the neutrino/sneutrino may cause sizable effects in the lightest Higgs boson mass. Such effects have an opposite sign to the top/stop effects and thus lighten the lightest Higgs boson. If the soft-breaking mass of the right-handed neutrino is sufficiently large, which can be realized in the split-SUSY scenario, the effects can lower the mass bound by a few tens of GeV.

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